Auto-adaptive algorithm using frequency derivatives of harmonic equations for automatic search of resonances

C. Dedeban, Jean-Pierre Damiano, P. Dubois, Jean-Paul Zolesio

AUTO-ADAPTATIVE ALGORITHM USING FREQUENCY DERIVATIVES OF HARMONIC EQUATIONS FOR AUTOMATIC SEARCH OF RESONANCES

C.Dedeban[*] J.P. Damiano [†] P.Dubois[‡] J.P. Zolésio [§]

September 30th 2005

Abstract

Electromagnetic modeling provides high accuracy but is often too time-consuming. Solving electromagnetic scattering and radiation problems with moment methods or finite element methods over a large frequency band requires the computer code to be run for every frequency sample. This is too expensive. So we propose a fast, accurate methodology for calculating the electromagnetic behaviour of an antenna with very few simulations over a large frequency band.

keywords: Electromagnetic, Integrals Equations, Frequency derivatives

1 Introduction

The computational electromagnetic simulation takes so long that the user often reduces the number of frequency samples in order to have a moderate

[*]`claude.dedeban@francetelecom.com` France Telecom R and D ANT/RES/TUR Fort de la tête de chien 06320 La Turbie

[†]`damiano@unice.fr` Laboratoires d'Electroniques, Antennes et Télecommunications, Université de Nice Sophia-Antipolis, 250 Albert Einstein06560 Valbonne

[‡]pierre.dubois@rd.francetelecom.com France Telecom R and D ANT/RES/TUR Fort de la tête de chien 06320 La Turbie

[§]jean-paul.zolesio@sophia.inria.fr CNRS and OPALE INRIA route des lucioles, 06560 Valbonne

computing time. However if the number of sample points is small, some physical effects may not be considered, so the obtained results are not right.

Today the cost of computation for the moment method is expensive because the computer model is running for every frequency sample. But from some years, various interpolation algorithms were developed and published ([1]-[5]). They were applied to build a moment matrix and its frequency interpolation.

Our aim is to know the current flows at the antenna surface over the frequency band using only a limited number of frequency samples adding specific information ([6],[7] and [8]) as the knowledge of the derivatives of these current flows. Then we developed an original adaptive polynomial interpolation. We present various results and comments for some structures.

2 Technical modeling

2.1 Analysis

We propose an original method taking into account the formal knowledge of the derivatives of the variational expressions of the current flows. It is numerically solved by a surface finite element method coupled with an interpolation adaptive algorithm . This approach takes into account the real electromagnetic behavior.

With the frequency derivability results associated to the Huyghens principle for C2 surfaces, and the two derivatives of the Rumsey reaction [10] obtained by a computer algebra system (Maple), we determined the unknown current flows and their derivatives at the antenna surface for a very small number of frequency samples ([7] and [8]). The expressions of the derivatives of the current flow become more complex when the order of derivation increases and the kernel singularity is never stronger than the original one. Nevertheless the integration of the successive kernels needs specific developments.

Solving numerically the Rumsey reaction, we use a finite element computer code (SR3D of France Tlcom R&D). This software is based on an integral equation formulation with a surface triangular finite element discretization as it is shown on the flowchart

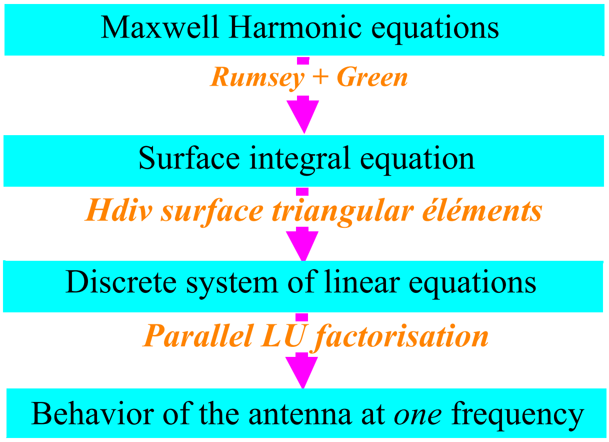


Figure 1 : Flow chart of the method

2.1.1 Surface integral equation

For a given ω we shearch in an homgeneous subdomain electric and magnetic fields of the form:

$$\begin{aligned}\vec{E}(\{\vec{J},\vec{m}\},x) &= \vec{E}^I_\omega(x) + i\omega\mu \oint_S G(\omega,x,y).\vec{J}(y)ds_y \dots \\ &-\frac{1}{i\omega\varepsilon}\oint_S \overrightarrow{grad_x} G(\omega,x,y).div_S\vec{J}(y)ds_y - \overrightarrow{rot}\left(\oint_S G(\omega,x,y).\vec{m}(y)ds_y\right)\end{aligned}$$

$$\begin{aligned}\vec{H}(\{\vec{J},\vec{m}\},x) &= \vec{H}^I_\omega(x) + i\omega\mu \oint_S G(\omega,x,y).\vec{m}(y)ds_y \dots \\ &-\frac{1}{i\omega\varepsilon}\oint_S \overrightarrow{grad_x} G(\omega,x,y).div_S\vec{m}(y)ds_y - \overrightarrow{rot}\left(\oint_S G(\omega,x,y).\vec{J}(y)ds_y\right)\end{aligned}$$

with $\vec{E}^I_\omega$,$\vec{H}^I_\omega$ the incident field, S the regular boundary of an homogeneous open set Ω, $\varepsilon\mu$ the material caracteristics and $\vec{j},\vec{m}$ the Electrical and magnetical currents density on S.

The Rumsey reaction between $\{\vec{J},\vec{m}\}$and $\{\vec{J}^t,\vec{m}^t\}$ is defined by

$$R_\omega\left(\{\vec{J},\vec{m}\},\{\vec{J}^t,\vec{m}^t\}\right) = \oint\left(\vec{E}(\{\vec{J},\vec{m}\},x)\cdot\vec{J}^t(x) - \vec{H}(\{\vec{J},\vec{m}\},x)\cdot\vec{m}^t(x)\right)ds_x \tag{1}$$

with N subdomains Ω_l limited by $S_l (l = 1, 2, \ldots, N)$ as shown above we can get a weak formulation of the classical harmonic Maxwell problem $\forall \{\vec{J}_l^t, \vec{m}_l^t\}$

$$\sum_{l=1}^{N} \left(R_\omega \left(\{\vec{J}_l, \vec{m}_l\}, \{\vec{J}_l^t, \vec{m}_l^t\} \right) - R_\omega \left(\{\vec{J}_l^I(\omega), \vec{m}_l^I(\omega)\}, \{\vec{J}_l^t, \vec{m}_l^t\} \right) \right) = 0$$

where $\{\vec{J}_l^I(\omega), \vec{m}_l^I(\omega)\}$ are the surface currents density associated to the incident field $\vec{E}_l^I, \vec{H}_l^I$ in Ω_l. If $u = (\{\vec{J}_l, \vec{m}_l\})$the equation can be written $\forall u^t \phi_\omega(u, u^t) = \phi_\omega(u^I(\omega), u^t)$ where $\phi_\omega$is a bilinear form. It can be written also $A(\omega, u) = B(\omega)$or $\Phi\left(\omega, u, u^I(\omega)\right) = 0$ so u is an implicit function of ω. From implicit function theorem due to the existence of the Colton-Kress isomorphism when $S_l$are C^2then $\frac{\partial u}{\partial \omega}$ exists.

2.1.2 Computing derivatives

By derivating this expression compared to the pulsation one finds :

$$\forall u^t \phi_\omega(u, u^t) = \phi_\omega(u^I(\omega), u^t) \Rightarrow \tag{2}$$

$$\forall u^t \phi_\omega(\frac{\partial}{\partial \omega} u, u^t) = \frac{\partial}{\partial \omega}(\phi_\omega(u^I(\omega), u^t)) - (\frac{\partial}{\partial \omega}\phi_\omega)(u, u^t) \tag{3}$$

also

$$A(\omega, u) = B(\omega) \Rightarrow (\frac{\partial}{\partial \omega} A)(\omega, u) + A\left(\omega, \frac{\partial}{\partial \omega} u\right) = \frac{\partial}{\partial \omega} B(\omega) \tag{4}$$

at the order k $\frac{\partial^k}{\partial \omega} u$ is solution of:

$$\phi_\omega(\frac{\partial^k}{\partial \omega} u, u^t) = \frac{\partial^k}{\partial \omega}(\phi_\omega(u^I(\omega), u^t)) - \sum_{m=1}^{k-1} C_k^m (\frac{\partial^{k-m}}{\partial \omega}\phi_\omega)(\frac{\partial^m}{\partial \omega} u, u^t) \tag{5}$$

Triangular surface finite elements are used to discretise these equations so to obtain the following scheme. where $A_h, u_h, B_h \ldots$are the restrictions in the finite element space of operator and solution.

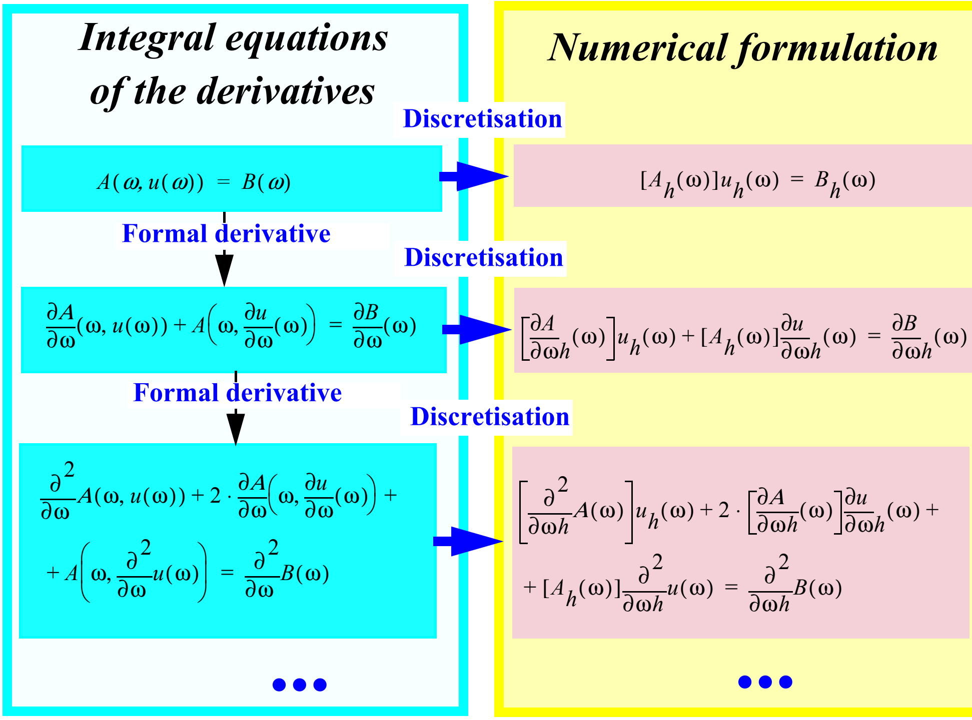


Figure 2 : Obtaining numerical expressions of the equations

We shall note $I \in C^n$ the components of $u_h$and A will be now the matrix of A_h.

For each frequency sample we have to solve a linear system for each derivative. Fortunately at one frequency the matrix of the linear systems are the same for all derivatives. So *we needs only one matrix factorisation for each frequency* as it can be shown in the flowchart below.

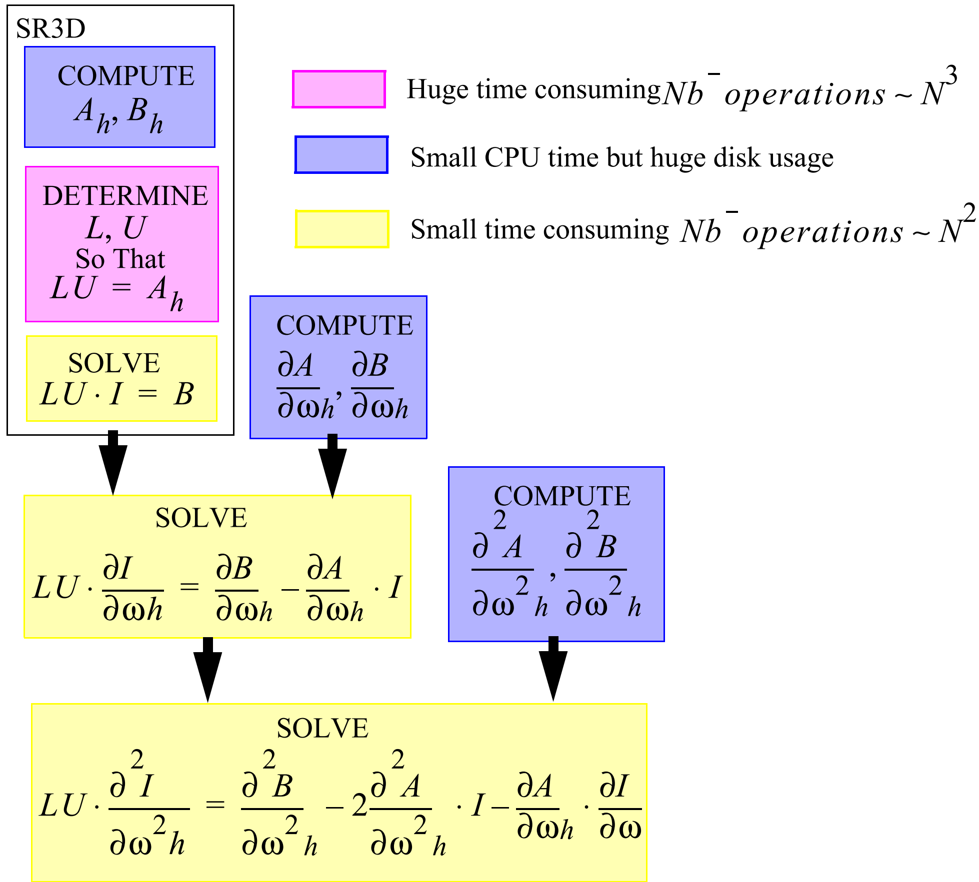


Figure 3 : Computing numerical values of current frequency derivatives

2.2 Interpolation

Once the successive derivatives of the unknown current flows are computed at some sampling points over the frequency band, our special adaptive interpolation routine is applied to evaluate them ([8]).

The analysis of the behavior of the interpolation function (interpolated current flows) allows to detect critical points where a small number of new sampling frequency points is needed. So the surface currents can be computed faster (ratio from 1 to 10) and we are able to deduce the expressions of the antenna characteristics.

antennas. However it appeers that in the case of a slotted line antenna the currents with the largest module were not well interpolated near resonance frequency of the patch as it is shown in Fig 4 below. If we add two frequency points correctly choosen the problem disapears.

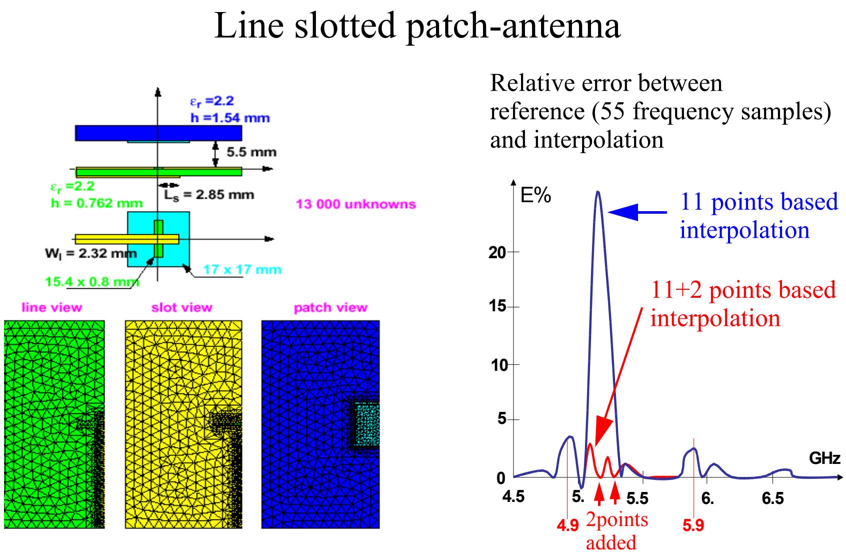


Fig 4 Improving accuracy

the average module of the M strongest discrete values of the current

This choice allows to improve the influence of resonances and mi

the number of frequency samples (figure 5).

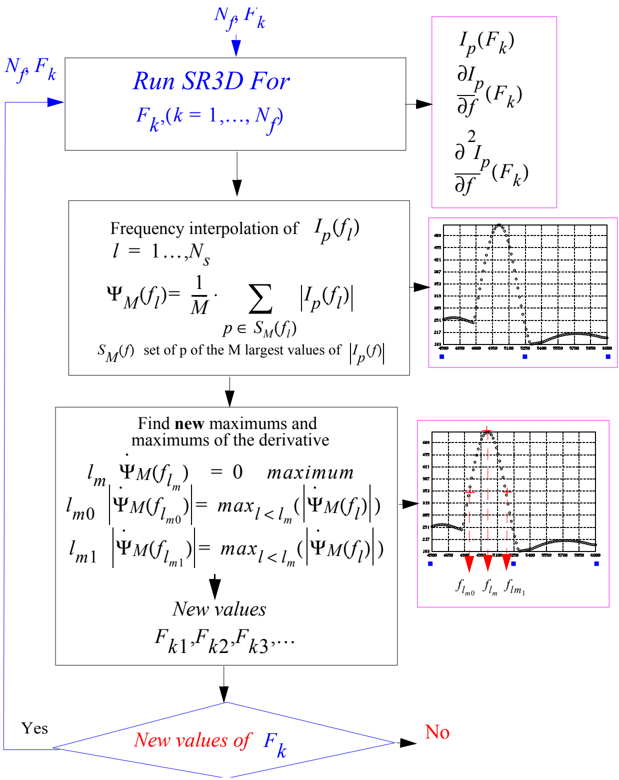


Fig 5 Autoadaptive algorithm

The figure (5) presents the variations of Ψ_M versus the frequency in the case of the line slotted patch antenna for successive levels of the autoadaptive process begining with three frequency points to show the robustness of the autoadaptive process. In this case a good precision is obtained on the full band width begining with 7 frequency samples. It is compared to the reference results of SR3D code with 55 frequency points.

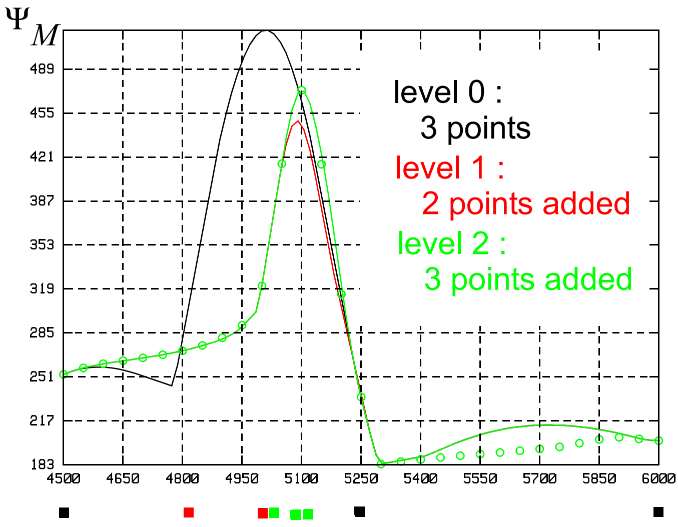


Fig 6 Auto adaptive process applied to the line slotted patch antenna

In each figure, we presents the comparison of the average of the twenty strongest values of the modulus of the current flow versus the frequency (4.5-6.0 GHz) .

3.2 Lossy resoning structure

As we succeed to catch the resonance frequency of a patch antenna we tried to study a dielectric cube starting with 10 frequency samples from 300Mhz to 2300Mhz.

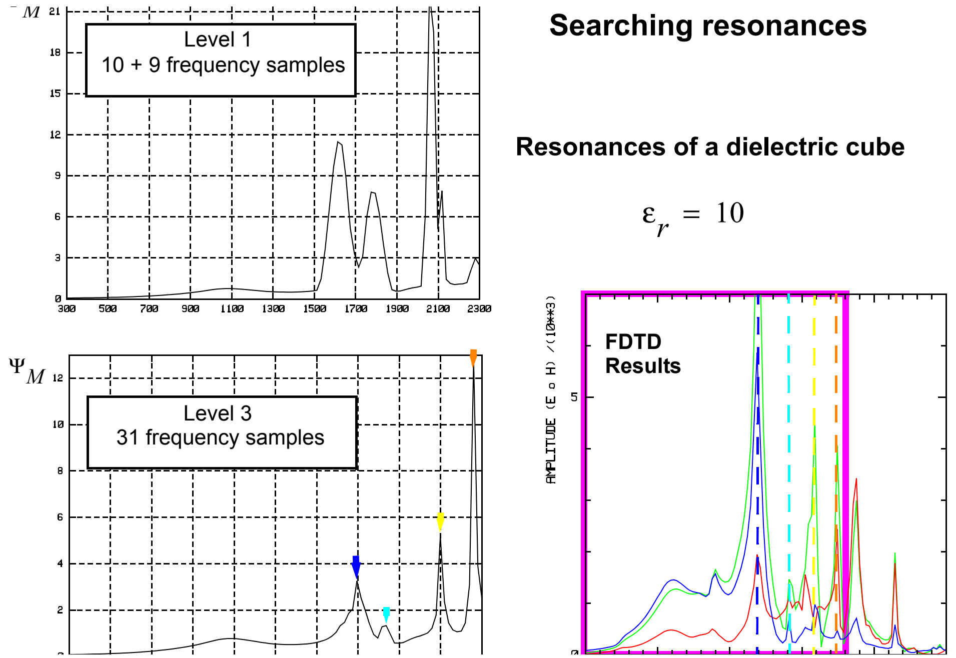
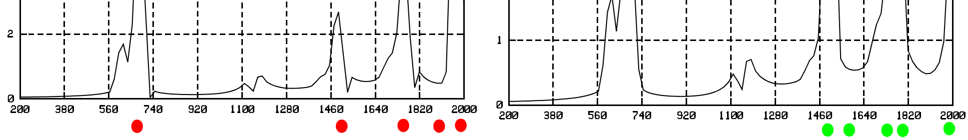


Fig 7 Autoadaptive search of resonance

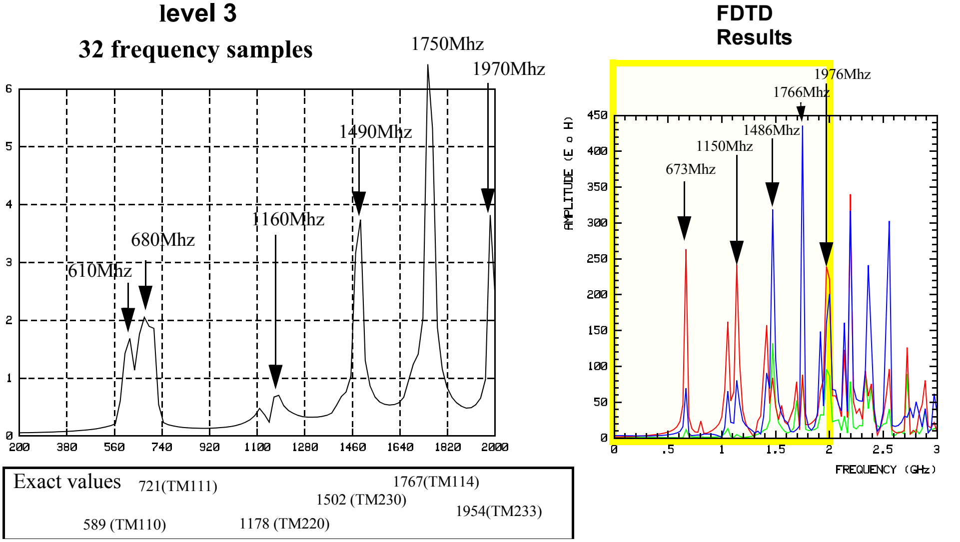
3.3 Searching resonance frequencies

The autoadaptive process was then applied to search resonance frequencies of a metallic cube the results were compared to the exact values of the cavity modes frequencies and to FDTD results



Resonances of a metallic cube

Final results



4 Conclusion

We presented an original and accurate auto adaptive technique to calculate the current flow at the antenna surface over a large frequency band, starting with a very small number of the frequency samples. We use the knowledge of the formal derivatives of the current flow associated with a piecewise polynomial interpolation. We compare our results with reference points obtained by a finite element code based on an integral-equation formulation without any optimization. We observe excellent results and an important computing time saving.

References

[1] J.C. Rautio, Planar Electromagnetic Analysis, *IEEE Microwave Magazine*,**3**, 35-41, 2003.

[2] J. Yeo, R. Mittra, An algorithm for interpolating frequency variations of MoM matrices arising in the analysis of planar microstrip structures, *IEEE Trans. MTT*, **51**, 3, 1018-1025, March 2003.

[3] X. Yang, E. Arvas, Use of frequency-derivative information in two-dimensional electromagnetic scattering problems*, IEE Proceedings, Part. H*, **138**, 4, 269-272, 1991.

[4] S. Narayana, T. K. Sarkar, R. Adve, M. Wicks, V. Vannicola A Comparison of Two Techniques for the Interpolation/Extrapolation of Frequency Domain Responses, *Digital Signal Processing*, **6**, 1, 51-67, 1996.

[5] B. Jiang, T. Su, H. Ling, Frequency Interpolation of Electromagnetic Scattering Data Using a Hybrid Model, *Microwave & Optical Technology Letters*, **27**, 5, 307-312, 2000.

[6] J.-P. Marmorat, J.-P. Zolsio, Rapport Contrat Armines-France Tlecom, septembre 2000.

[7] P. Dubois, J.P. Damiano, C. Dedeban, J.P. Zolesio, Applications de la drivation formelle en frquence d'une formulation intgrale base sur la raction de Rumsey pour l'tude de sources, *Proceedings of the Journes Internationales de Nice sur les Antennes (JINA),* 12-14 nov. 2002, Nice, France, vol.1, 139-142.

[8] W. Koepf, Efficient Computation of Chebyshev Polynomials *in Computer Algebra Systems: A Practical Guide* (Ed. M. J. Wester). New York: Wiley, 79-99, 1999.

[9] C. Brezinski, *Pad-Type Approximation and General Orthogonal Polynomials*, Birkhuser, Basel, 1980.

[10] P. Liardet, P. Stambul, Algebraic Computation with Continued Fractions. *J. Number Theory*, **73**, 92-121, 1998.